

Resolving the singularity of the Hawking-Turok type instanton

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Abstract

We point out that the singular instanton of Hawking-Turok type, in which the singularity occurs due to the divergence of a massless scalar field, can be generated by Euclideanized regular p -brane solutions in string (or M-) theory upon compactification to four dimensions.

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I. INTRODUCTION

Hawking and Turok(HT) have recently discovered an instanton for the creation of an open universe from nothing [1]. This leads to an interesting possibility of realization of open inflation with more realistic form of the inflaton potential. However, the HT-instanton contains a genuine singularity: both the curvature and the scalar field diverge there. Although the action is finite, further justification for allowing such a singular instanton is necessary. In fact, as Vilenkin has shown [2], instantons with the same singularity structure as that of HT-instanton lead to unacceptable physical consequences: the decay of flat space.

Garriga has recently shown [3] that the singularity can be resolved in five dimensions and that flat space with a compact extra dimension can be sufficiently long lived if the size of the extra dimension is large compared with the Planck length in four dimensions (see also [4]). In fact, the fundamental energy scale in M-theory [5] is the eleven-dimensional Planck mass $M_{11} \simeq 10^{17}\text{GeV}$ [6,7]. Unfortunately, Garriga's solution is not derived in the context of string(or M-)theory. On the other hand, Larsen and Wilczek have shown [8] that a class of (3+1)-dimensional FRW cosmologies can be embedded within a variety of solutions of string theory.

In this note, being motivated by Larsen and Wilczek's approach, we show that the instanton with the same singularity structure can be generated by Euclideanized regular p -brane solutions in string theory (or eleven-dimensional supergravity) upon compactification to four dimensions.

II. HIGHER-DIMENSIONAL RESOLUTION OF COSMOLOGICAL SINGULARITIES

A. from five to four

Let us briefly review the procedure by which Larsen-Wilczek embed a FRW universe model in a non-singular fashion. They started the discussion by reference to the five-

dimensional interior Schwarzschild metric as an example. It can be written as

$$ds^2 = e^{-4\sigma} dy^2 - e^{4\sigma} d\tau^2 + \tau^2 d\Omega_3^2, \quad (1)$$

$$e^{-4\sigma} = \frac{\mu}{\tau^2} - 1, \quad (2)$$

where $\mu > 0$ is a mass parameter. The range of τ is $0 < \tau^2 < \mu$. Compactification along y induces an effective four-dimensional dilaton $e^{-2\phi_4} = e^{-2\sigma}$ with the string frame metric $g_{S\mu\nu}$ in which the dy^2 term is omitted in Eq.(1). In terms of the Einstein frame metric $g_{E\mu\nu} = e^{-2\phi_4} g_{S\mu\nu}$, the four-dimensional cosmology becomes

$$ds_E^2 = -e^{2\sigma} d\tau^2 + e^{-2\sigma} \tau^2 d\Omega_3^2. \quad (3)$$

We shall work in the Einstein frame line element because it is invariant under the standard duality transformations in the string theory context and because a test particle follow geodesic in this frame [8]. Since in the Einstein frame the effective four-dimensional action is that of a massless scalar, minimally coupled to gravity, the four-dimensional FRW cosmology is the closed model with matter of $\rho = p$ and singularities at both the big-bang and the big-crunch.

Thus, the five dimensional vacuum black hole solution with a coordinate singularity (the horizon) and a physical singularity (the origin) becomes four dimensional cosmology with a massless scalar field and physical singularities at both initial and final times.

We note here that Garriga's five-dimensional regular instanton is nothing but the five-dimensional Schwarzschild instanton. To show that, we introduce the time coordinate t by

$$dt = e^{2\sigma} d\tau, \quad (4)$$

so that $t^2 = \mu - \tau^2$.¹ Then Eq.(1) becomes

¹Here the integration constant is neglected.

$$ds^2 = -dt^2 + \tau^2 d\Omega_3^2 + e^{-4\sigma} dy^2, \quad (5)$$

$$= -dt^2 + (\mu - t^2)d\Omega_3^2 + \frac{t^2}{\mu - t^2} dy^2. \quad (6)$$

Thus, if we complexify t and y such that

$$T = it, \quad Y = iy \quad (7)$$

with T and Y being real, then Eq.(6) is Euclideanized as

$$ds^2 = dT^2 + (\mu + T^2)d\Omega_3^2 + \frac{T^2}{\mu + T^2} dY^2, \quad (8)$$

which is the same as the Garriga's regular five-dimensional instanton solution² (if $0 \leq Y \leq 2\pi\mu^{1/2}$) and solves the Euclidean field equations for pure gravity in five dimensions [3]. Changing into the Einstein frame after compactifying Y yields the following metric and massless scalar field:

$$ds^2 = \frac{T}{\sqrt{\mu + T^2}} \left(dT^2 + (\mu + T^2)d\Omega_3^2 \right) \quad (9)$$

$$\equiv d\bar{T}^2 + b(\bar{T})^2 d\Omega_3^2, \quad (10)$$

$$\phi \propto -\ln \left(\frac{T^2}{\mu + T^2} \right). \quad (11)$$

The internal radius of the fifth dimension can be zero, but it is just a coordinate singularity. However, that zero-point induces a singularity of the Einstein frame metric in four dimensions. In that way, the singularity in the four-dimensional world is induced. Near the singularity $T \simeq 0$, $\bar{T} - \bar{T}_f \simeq 2\mu^{-1/4}T^{3/2}/3$ with \bar{T}_f being a constant. Thus the scale factor $b(\bar{T})$ and the scalar field behave as

$$b(\bar{T}) \simeq \mu^{1/4}T^{1/2} \propto |\bar{T} - \bar{T}_f|^{1/3}, \quad (12)$$

$$\phi \propto -\ln T + \text{const.} \propto -\ln |\bar{T} - \bar{T}_f| + \text{const.} \quad (13)$$

This is the same singularity structure as that of the Hawking-Turok(or Vilenkin) instanton [1–3].

²Garriga's instanton can also be obtained from the exterior Schwarzschild metric.

B. from ten to four

Larsen-Wilczek further generalized the model and embedded the cosmology in a solution that originates from string theory.

The starting point is the classical solution associated with the black p -brane [9–11]. In the string frame it is written as

$$ds_S^2 = e^{2\xi}(e^{-4\sigma}dy^2 + dx_1^2 + \dots + dx_p^2) + e^{-2\xi}(dx_{p+1}^2 + \dots + dx_5^2) + e^{-2\xi}(-e^{4\sigma}d\tau^2 + \tau^2 d\Omega_3^2), \quad (14)$$

$$e^{-2\phi_{10}} = e^{-2\xi(p-3)}, \quad (15)$$

$$F_{p+2} = \partial_\mu e^{4\xi} dy \wedge dx_1 \wedge y \dots \wedge dx_p \wedge dx^\mu, \quad (16)$$

where F_{p+2} is the $(p+2)$ -form Ramond-Ramond field strength and ξ and σ are defined as

$$e^{-4\xi} = \frac{q}{\tau^2} + 1, \quad (17)$$

$$e^{-4\sigma} = \frac{\mu}{\tau^2} - 1. \quad (18)$$

Compactification to four-dimensions³ induces an effective dilaton $e^{-2\phi_4} = e^{2\xi-2\sigma}$ independent of p . Remarkably, the four dimensional Einstein metric is again Eq.(3). Thus the black p -brane generates the same four-dimensional cosmology as the five-dimensional Schwarzschild solution does.

The corresponding Euclidean solution can be immediately obtained by following the same procedure (4) and (7) made in five-dimensional case. Namely, we have

$$ds_S^2 = \left(\frac{\mu + T^2}{q + \mu + T^2}\right)^{1/2} \left(\frac{T^2}{\mu + T^2} dY^2 + dx_1^2 + \dots + dx_p^2\right) + \left(\frac{\mu + T^2}{q + \mu + T^2}\right)^{-1/2} (dx_{p+1}^2 + \dots + dx_5^2) + \left(\frac{\mu + T^2}{q + \mu + T^2}\right)^{-1/2} (dT^2 + (\mu + T^2)d\Omega_3^2). \quad (19)$$

This solution is again regular if $0 \leq Y \leq 2\pi((q + \mu)\mu)^{1/4}$.

³Here we only consider toroidal compactification. For sphere compactification, see [12].

C. from ten to eleven

One solution in Type IIA string theory is derived from another in eleven-dimensional supergravity by compactifying on a circle [13–15]:

$$\exp(4\phi_{10}/3) = g_{M11,11}, \quad (20)$$

$$g_{S\mu\nu} = \exp(2\phi_{10}/3)g_{M\mu\nu}, \quad (21)$$

where $g_{M\mu\nu}$ is M-theory frame metric. Conversely, one solution in Type IIA string theory is “oxidated” to another solution in eleven-dimensional supergravity. For example, black five-brane solution in eleven-dimensional supergravity can be obtained from black four-brane in string theory:

$$\begin{aligned} ds_{11}^2 = & e^{4\xi/3}(e^{-4\sigma}dy^2 + dx_1^2 + \dots + dx_5^2) + e^{-2\xi}dx_6^2 \\ & + e^{-2\xi}(-e^{4\sigma}d\tau^2 + \tau^2d\Omega_3^2). \end{aligned} \quad (22)$$

We also get black two-brane in eleven dimensions from black two-brane in ten dimensions:

$$\begin{aligned} ds_{11}^2 = & e^{8\xi/3}(e^{-4\sigma}dy^2 + dx_1^2 + dx_2^2) + e^{-4\xi/3}(dx_3^2 + \dots + dx_6^2) \\ & + e^{-4\xi/3}(-e^{4\sigma}d\tau^2 + \tau^2d\Omega_3^2). \end{aligned} \quad (23)$$

Corresponding Euclidean solutions can be obtained in the same manner.

III. SUMMARY

We have shown that the singularity of the instanton of Hawking-Turok type can be resolved from higher-dimensional viewpoint within the context of M-(or string) theory. We have also shown that Garriga’s instanton is nothing but the five-dimensional Schwarzschild instanton. The same singularity structure emerges when a regular instanton in higher dimensions is compactified to four dimensions. The fundamental length scale in M-theory is much larger than the four dimensional Planck length, and therefore the decay of flat space is strongly suppressed.

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